<u>Perturbation Methods for Nonlinear Continuous Dynamical Systems in Engineering.</u>

MSc/PhD course: 3 ECTS, 10 lectures of 2 hours each, 40 hours for self-study, home work and oral exam.

<u>Lecture 1:</u> Introduction to the theory of stability. Stability of equilibrium solutions, and stability of periodic solutions. Linearisation. Asymptotic stablity and Lyapunov stability. Stability analysis of linear equations with variable coefficients.

<u>Lecture 2:</u> Floquet theory, Stability by linearisation. Blow-up techniques. Introduction to perturbation theory. Naive expansions, asymptotic expansions. The Poincaré expansion theorem.

<u>Lecture 3:</u> Introduction to the multiple time-scales perturbation method for ordinary differential equations. Mathematical justification of the method. Application of the method to linear and nonlinear ordinary differential equations. Raleigh and Van der Pol equation.

<u>Lecture 4:</u> Introduction to the multiple scales perturbation method for partial differential equations. Mathematical justification of the method. Application of the method to linear and nonlinear partial differential equations. Vibrations of a string-like structure on a nonlinear elastic foundation. Telegraph and Klein-Gordon equations.

<u>Lecture 5:</u> Application of the multiple scales perturbation method to beam-like structures. Weakly nonlinear vibrations of beams. Applicability of the Galerkin truncation method to string-like and beam-like problems. Resonance conditions and the small denominator problem.

<u>Lecture 6:</u> When Galerkin's truncation method is not applicable to string-like problems to obtain accurate approximations on long time-scales, it will be shown how the multiple time-scales perturbation method in combination with the method of characteristic coordinates can be used in some cases to obtain accurate results on long time-scales. The method is applied to conveyor belt problems and to elevator cable vibrations. Theory for first order partial differential equations: method of characteristics.

<u>Lecture 7:</u> Outline of the mathematical justification of the applied methods to wave and beam equations. Several examples in the field of linear and nonlinear vibrations of elastic structures (such as strings, beams, and plates) will be given to show the restrictions of the applicability of Galerkin's truncation method. It will be made clear how infinite dimensional systems of ordinary differential equations might be studied.

<u>Lecture 8:</u> Introduction to singular perturbation theory for ordinary differential equations. Boundary layers and interior layers. Multiple time-scales perturbation method and the WKBJ-method.

Lecture 9: The averaging method. The Lagrange standard form. Averaging in the periodic case. Averaging in the general case. Adiabatic invariants. Resonance manifolds. Periodic solutions.

Lecture 10: The aim of this lecture is to study autoresonance phenomena in a space-time-varying mechanical system. The maximal amplitude of the autoresonant solution and the time of autoresonant growth of the amplitude of the modes of fast oscillations are determined. A vertically translating string with a time-varying length and a space-time-varying tension are considered. The problem can be used as a simple model to describe transversal vibrations of an elevator cable for which the length changes linearly in time. It is assumed that the axial velocity of the cable is small compared to nominal wave velocity and that the cable mass is small compared to car mass. The elevator cable is excited sinusoidally at the upper end by the displacement of the building in the horizontal direction from its equilibrium position caused by wind forces. This external excitation has a constant amplitude of order ε. It is shown that order ε amplitude excitations at the upper end result in order $\varepsilon^{1/2}$ solution responses. Interior layer analysis has been provided systematically to show that there exists an unexpected timescale of order $\varepsilon^{-1/2}$. For this reason, a three-timescale perturbation method is used to construct asymptotic approximations of the solutions of the initial-boundary value problem.

Home-work and oral exam: During the course, the student has to make homework exercises. Halfway and at the end of the course, the student has to hand in the home work by e-mail. The home work will be graded and counts for 50% in the final grading. The other 50% is determined by an oral exam which will be held in the last week of November. Announcements about these oral exams will be given in November.

Literature:

[1]. F. Verhulst, Nonlinear Differential Equations and Dynamical Systems, 2nd edition, Springer-Verlag, 1996.
[2]. A.H. Nayfeh, and D.T. Mook, Nonlinear Oscillations, Wiley-Interscience, 1979.
[3]. W.T. van Horssen, and A.H.P. van der Burgh, On initial boundary value problems for weakly semilinear telegraph equations. SIAM J. Appl.Math. 48 (1988), p. 719-736.
[4] W.T. van Horssen, An asymptotic theory for a class of initial-boundary value problems for weakly nonlinear wave equations, SIAM J. Appl. 48 (1988), p. 1227-1243.
[5] G.J. Boertjens, and W.T. van Horssen, On interactions of oscillation modes for a weakly nonlinear beam with an external force, J. of Sound and Vibration 235 (2000), p.201-217.
[6] G.J. Boertjens, and W.T. van Horssen, On mode interactions for a weakly nonlinear beam equation, Nonlinear Dynamics 17 (1998), p.23-40.
[7] R.A. Malookani, and W.T. van Horssen, On the asymptotic approximation of the solution of an equation for a non-constant axially moving string, J. of Sound and Vibration 367 (2016), p.203-218.
[8] S.H. Sandilo, and W.T. van Horssen, On a cascade of auto resonances in an elevator cable system, Nonlinear Dynamics 80 (2015), p.1613-1630.